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The Infrared Behavior of QCD Propagators in Landau Gauge*

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Some features of the solutions to the truncated Dyson-Schwinger equations(DSEs) for the propagators of QCD in Landau gauge are summarized. In particular, the Kugo-Ojima confinement criterion is realized, and positivity of transverse gluons is manifestly violated in these solutions. In Landau gauge, the gluon-ghost vertex function offers a convenient possibility to define a nonperturbative running coupling. The infrared fixed point obtained from this coupling which determines the 2-point interactions of color-octet quark currents implies the existence of unphysical massless states which are necessary to escape the cluster decomposition of colored clusters. The gluon and ghost propagators, and the nonperturbative running coupling, are compared to recent lattice simulations. A significant deviation of the running coupling from the infrared behavior extracted in simulations of 3-point functions is attributed to an inconsistency of asymmetric subtraction schemes due to a consequence of the Kugo-Ojima criterion: infrared enhanced ghosts.

1. Confinement in Landau Gauge QCD

Covariant quantum theories of gauge fields require indefinite metric spaces. Modifications to the standard (axiomatic) framework of quantum field theory are also necessary to accommodate confinement in QCD. These seem to be given by the choice of either relaxing the principle of locality or abandoning the positivity of the representation space. Great emphasis has therefore been put on the idea of relating confinement to the violation of positivity in QCD. Just as in QED, where the Gupta-Bleuler prescription is to enforce the Lorentz condition on physical states, a semi-definite *physical subspace* can be defined as the kernel of an operator. The physical states then correspond to equivalence classes of states in this subspace differing by zero norm components. Besides transverse photons covariance implies the existence of longitudinal and scalar photons in QED. The latter two form metric partners in the indefinite space. The Lorentz condition eliminates half of these leaving unpaired states of zero norm which do not contribute to observables. Since the Lorentz condition commutes with the S-Matrix, physical states scatter into physical ones exclusively. Color confinement in QCD is ascribed to an analogous mechanism: No colored states should be present in the positive definite space of physical states defined by some suitable condition maintaining physical S-matrix unitarity. Within the framework of

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BRS-algebra the completeness of the nilpotent BRS-charge Q_B in a state space $\mathcal{V}$ of indefinite metric is assumed. The (semi-definite) physical subspace $\mathcal{V}_{\text{phys}} = \operatorname{Ker} Q_B$ is defined by those states which are annihilated by the BRS charge Q_B. Positivity is then proved for physical states [1] which are given by the cohomology $\mathcal{H}(Q_B, \mathcal{V}) = \operatorname{Ker} Q_B / \mathrm{Im} Q_B \simeq \mathcal{V}_s$, the covariant space of equivalence classes of BRS-closed modulo exact states (in the image $\mathrm{Im} Q_B$ of the BRS-charge).

In perturbation theory the space $\mathcal{H}$ is formed by transverse gluon and quark states. Longitudinal and timelike gluons form (massless) quartet representations with ghosts and are thus unphysical. At the same time the global symmetry $J^a_{\mu,\nu}$ corresponding to gauge transformations generated by $\theta^a(x) = a^a_\mu x^\mu$ is spontaneously broken quite analogous to the displacement symmetry in QED. Nonperturbatively, Kugo and Ojima have shown that the identification of BRS-singlet states (in $\mathcal{V}_s$) with color singlets is generally possible, in particular also for transverse gluons and quarks, if this global symmetry is restored dynamically [1–3]. A sufficient condition for this restoration to happen is that the nonperturbative ghost propagator is more singular than a massless pole in the infrared, *i.e.*,

$$G(p) = \frac{-1}{p^2(1+u(p^2))}\,, \quad \text{with} \quad u(p^2) \to -1 \ \text{ for } \ p^2 \to 0\,. \tag{1}$$

Furthermore, this mechanism is related to Nishijima's derivation [4], from Ward-Takahashi identities, of the Oehme-Zimmermann superconvergence relations which formalize a long known contradiction between asymptotic freedom and the positivity of the spectral density for transverse gluons in the covariant gauge [5].

The remaining dynamical aspect of confinement in this formulation resides in the cluster decomposition property of local field theory. The proof of which, absolutely general otherwise, does not include the indefinite metric spaces of covariant gauge theories [6]. In fact, there is quite convincing evidence for the contrary, namely that the cluster property does not hold for colored correlations of QCD in such a description [7]. This would thus eliminate the possibility of scattering a physical state into color singlet states consisting of widely separated colored clusters (the "behind-the-moon" problem, see also Ref. [2] and references therein). Then, however, there cannot be a mass gap in the whole indefinite space $\mathcal{V}$ (which implies nothing on the physical spectrum of the mass operator in $\mathcal{H}$).

Our solutions to the DSEs for the ghost and gluon propagators as reported in Refs. [8] provide for both, the Kugo-Ojima confinement criterion by an infrared enhancement of the ghost propagator and the gapless spectrum by the infrared fixed point in the color-octet correlations. The violation of positivity of transverse gluons observed in [8] seems unambiguously established by a variety of independent nonperturbative studies of the gluon propagator as summarized in Ref. [9]. It is furthermore interesting to note that recent lattice calculations also verify the Kugo-Ojima criterion [10].

2. Infrared Behavior of Gluon and Ghost Propagators

The known structures in the 3-point vertex functions can be employed to establish truncated DSEs that are complete for the gluon, ghost and quark propagators of Landau gauge QCD [8,11]. This is possible with systematically neglecting contributions from explicit 4-point vertices to the propagator DSEs as well as non-trivial 4-point scattering kernels in the constructions of the 3-point vertices. The coupled DSEs for the propagators of

the pure gauge theory can be solved in a one-dimensional approximation [8]. Asymptotic expansions of the solutions in the infrared are obtained analytically. While the gluon propagator is found to vanish for small spacelike momenta in this way, an apparent contradiction with earlier studies that implied its infrared enhancement can be understood from the observation that the previously neglected ghost propagator now assumes just this: An infrared enhancement of ghosts as predicted by the Kugo-Ojima confinement criterion [3]. This infrared behavior of the propagators in Landau gauge was later confirmed qualitatively by studies of further truncated DSEs [12]. Neither does it thus seem to depend on the particular 3-point vertices nor on the one-dimensional approximation employed in our original solutions.

These solutions to the coupled gluon-ghost DSEs compare well with recent lattice results available for the gluon [13,14] and the ghost propagator [15] (which implement various lattice versions of the Landau gauge condition). Indications towards an infrared vanishing gluon propagator are now seen on the lattice also [14]. At least some suppression in the infrared is certainly excluded to be a finite size effect [13]. Especially the ghost propagator, however, is in compelling agreement with the lattice data. This is an interesting result for yet another reason: In [15,14] the Landau gauge condition was supplemented by an algorithm to select gauge field configurations from the fundamental modular region which is to avoid Gribov copies. Thus, our results suggest that the existence of such gauge copies might have little effect on the solutions to the Landau gauge DSEs. The positivity violations of transverse gluons as seen in our results [8] combined with the evidence from roughly a decade of lattice simulations with considerably increasing statistics [16,9] leave little doubt on the significance of this result.

3. Nonperturbative Running Couplings

Lattice Landau gauge results have also become available for nonperturbative running couplings from simulations of the 3-gluon [17,18] and the quark-gluon vertex [19]. These seem to have the common feature of a maximum value $\alpha_S^{\rm max}(\mu_0)$ at a *finite* momentum scale $\mu_0 > 0$. Below this scale decreasing values of the couplings are extracted towards smaller scales. Such qualitative forms lead to double-valued β-functions, however. Here, we would like to point out that this behavior seems likely to be an artifact of asymmetric subtraction schemes in theories with confinement realized by the Kugo-Ojima criterion. The reason is seen by relating results from asymmetric schemes $\alpha_S^{\rm asym}$ to those of symmetric subtraction schemes $\alpha_S^{\rm sym}$ which essentially results in a ratio of ghost renormalization functions [11],

$$\alpha_S^{\rm asym}(\mu) \propto \lim_{s\to 0} \frac{1+u(\mu^2)}{1+u(s)} \, \alpha_S^{\rm sym}(\mu) \; . \qquad (2)$$

With $u(0) = -1$ this is not well-defined. If the attention is limited to the μ-dependence of $\alpha_S^{\rm asym}$, however, the Kugo-Ojima criterion implies an artificial suppression in $\alpha_S^{\rm asym}(\mu)$ as compared to $\alpha_S^{\rm sym}(\mu)$.

The (symmetric) scheme of [8] is based on the non-renormalization of the ghost-gluon vertex in Landau gauge. The ideal comparison would thus be provided by a lattice simulation employing this vertex to extract the running coupling in a likewise symmetric subtraction scheme. It would be very interesting to see whether such simulations could

support the infrared fixed point we obtained from the DSEs in [8] (with $\alpha_S(\mu \to 0) \approx 9.5$ and monotonically decreasing for $\mu > 0$). This is because such an infrared fixed point entails the existence of unphysical massless (bound) states in the color-octet channels of 4-point functions (of gluon/ghost and quark/antiquark correlations). It might also be worthwhile to consider simulations of such 4-point functions in lattice Landau gauge, and to assess possible indications for massless states in these correlations directly. The existence of massless unphysical states is a necessary condition for a failure of the cluster decomposition property for colored clusters, see Chap. 4.3.4 in Ref. [2].

Recently we solved the coupled propagator DSEs with quarks included [20]. Implications on positivity violations and confinement for quarks are currently being investigated.

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